Causality

- Focused on associative relationships
  - → How do college graduates differ from non-college graduates in terms of income
  - → Predict the weather
    - (Not interested in changing the world)
    - Predict a mudslide

- Causal Relationships
  - → What would happen to unemployment if we raised the minimum wage?
  - → What would prices look like if ATT + Verizon merged?
  - → What is the effect of having a "good" teacher on educational attainment?

    - → Some hypothetical world in mind

Problems w/ Causality from Observational Data

1) Causality. What is it?

- "What is a causal relationship"?
- Heterogeneity is causal relationships
- Need a model of causality / formal way of think

"Causality comes from a model"

2) What causes what?

Suppose I see that people that take advil have lower headaches?

↓ What is causing what.

- Reverse Causality Issue

3. Confounding Variable / Select into treatment

→ What is effect of UCLA on education outcomes?

↓

UCLA have to be admitted

- Ability that is affecting both their treatment outcome & also affecting later life outcome

↓

What is the effect of vaccination on childhood to catch COVID.

→ People who get vaccinated have high trust in CDC

→ May be more likely to follow social distancing & masking guidelines

Problem 1: What is Causality?

→ Develop a model of causality

"Potential Outcomes Framework"

→ Medicine & Economics

→ Rubin Potential Outcomes

Do-Calculus

→ CS & ML

Judea Pearl (UCLA)

Suppose we have $D \in \{0, 1\}$ — College / UCLA

Want to know the effect of D on some outcome Y

Income at age 40

Every person has two "potential" outcomes

Fundamentally an Distribution of Y(1) & Y(0)

Y(1) — Outcome if you take up treatment (D=1)

→ Income if you go to college

Y(0) — Outcome if you don't take up treatment (D=0)

→ Income if you don't go to college

$\mathbb{E}[Y(1)]$ — Average income if every unit to college

$\mathbb{E}[Y(0)]$ — Average income if no one went to college

$\mathbb{E}[Y(1) - Y(0)]$ = Average Treatment effect

$Pr(Y(1) \le -10{,}000)$

These are all things we could compute if we know distribution

Problem: Do not observe $Y(1)$ or $Y(0)$, only observe Y - realized outcome

$$Y = D\,Y(1) + (1-D)\,Y(0)$$

$$Y = \begin{cases} Y(1) & \text{if } D=1 \\ Y(0) & \text{if } D=0 \end{cases}$$

$$D \not\perp (Y(1), Y(0))$$

$E[Y(1)]$

Try $\quad \underline{E[Y \mid D=1]} = E[\overset{1}{D}\cdot Y(1) + (1-\overset{1}{D})\,Y(0) \mid D=1]$

$$= E[Y(1) \mid D=1]$$

$$\neq E[Y(1)]$$

Identification Issue, can't identify $E[Y(1)]$ from joint distribution of Y & D.

1) First Note

Potential Outcomes Depend only on own treatment

→ "SUTVA"

2) Model

3) Directions of Causality come from Background Knowledge

$$D \not\perp (Y(1), Y(0))$$

Now how do we deal with this?

<u>Problem:</u> Cannot Experiment

<u>Experiment:</u> $D \perp (Y(1), Y(0))$

$$E[Y \mid D=1] = E[Y(1) \mid D=1]$$

can drop

$$= E[Y(1)]$$

2 Strategies

→ Conditional Independence

→ Most of us

→ Instrumental Variables / LATE

→ Imbens & Angrist & Rubin (1996, JASA)

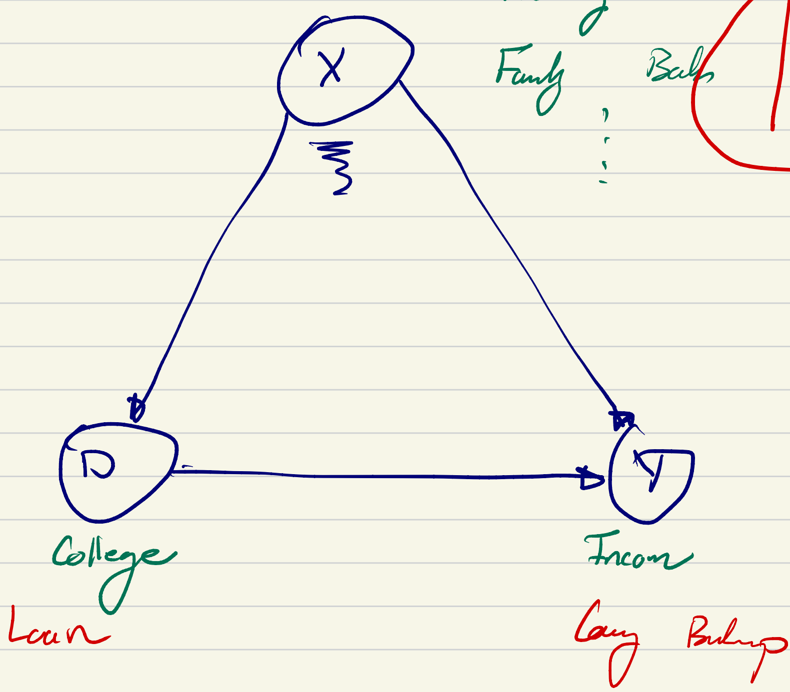
$$\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y|X]]$$

Conditional Independence

$$D \not\perp (Y(1), Y(0))$$

Ability
Family
Background
⋮

Age, Credit, Sex
⋮



- Suppose I observe all my confounding variables
- Just condition to get counterfactual

$$\boxed{Y(1), Y(0) \perp D \mid X}$$

$E[Y(1)]$

$$E[Y \mid D=1, X] = E[Y(1) \mid D=1, X]$$

$$= \underline{E[Y(1) \mid X]}$$

Average over X to get $\mathbb{E}[Y(1)]$

Instrumetal Variable Analysis

- Effect of therapy on Anxiety levels

  $D \in \{0, 1\}$ whether or not you go to therapy

  $Y(1) \leftarrow$ Your potential anxiety level if go to therapy

  $Y(0) \leftarrow$ Your potential anxiety level if you don't go to therapy

- Cannot experiment

- $D \not\perp (Y(1), Y(0))$

- Suppose we randomly offer people a voucher for going to therapy

  $Z \in \{0, 1\}$

  Don't receive voucher — Do receive the voucher

  $Z \perp (Y(1), Y(0))$

  $Cov(Z, D) \neq 0$

4 Types of People

	Always Takers	Never Takers	Compliers	~~Defiers~~
Receive Voucher	$D=1$	$D=0$	$D=1$	~~$D=0$~~
Don't Receive Voucher	$D=1$	$D=0$	$D=0$	~~$D=1$~~

Assume no Defiers

$$\Pr(D=1 \mid Z=1)$$

$$= \Pr(\text{Always Taker}) + \Pr(\text{Complier})$$

$$\Pr(D=1 \mid Z=0)$$

$$= \Pr(\text{Always Taker})$$

$$\Rightarrow \underline{\Pr(D=1 \mid Z=1)} - \underline{\Pr(D=1 \mid Z=0)}$$

$$= \Pr(\text{Compliers})$$

$$\mathbb{E}[YD \mid Z=1] = \mathbb{E}[Y(1) \mid Z=1, \text{Always Taker}] \cdot \Pr(\text{Always Taker})$$

$$+ \ \mathbb{E}[Y(1) \mid Z=1, \text{Complier}] \cdot \Pr(\text{Complier})$$

$$= \mathbb{E}[Y(1) \mid \text{Always Taker}] \cdot \Pr(\text{Always Taker})$$

$$+ \ \mathbb{E}[Y(1) \mid \text{Complier}] \cdot \Pr(\text{Complier})$$

$$\mathbb{E}[YD \mid Z=0] = \mathbb{E}[Y(1) \mid Z=0, \text{Always Taker}] \cdot \Pr(\text{Always Taker})$$

$$= \mathbb{E}[Y(1) \mid \underline{\text{Always Taker}}] \cdot \Pr(\text{Always Taker})$$

$$\mathbb{E}[YD \mid Z=1] - \mathbb{E}[YD \mid Z=0] = \mathbb{E}[Y(1) \mid \text{Complier}] \cdot \Pr(\text{Complier})$$

$$\frac{\mathbb{E}[YD \mid Z=1] - \mathbb{E}[YD \mid Z=0]}{\Pr(D=1 \mid Z=1) - \Pr(D=1 \mid Z=0)} = \frac{\mathbb{E}[Y(1) \mid \text{Complier}] \cdot \Pr(\text{Complier})}{\Pr(\text{Complier})}$$

N

D

$$= \mathbb{E}[Y(1) \mid \text{Complier}]$$

How do we estimate these quantities?

$$\{Y_i, D_i, Z_i\}_{i=1}^{n}$$

Recall from before that if $X \in \{0,1\}$

$$Y = \beta_0 + \beta_1 \cdot X + \varepsilon$$

$$\beta_1 = \underline{\mathbb{E}[Y \mid X=1]} - \underline{\mathbb{E}[Y \mid X=0]}$$

$\hat{\beta}_1$

To estimate N: Regress YD on $Z \in \{0,1\}$ and look at $\hat{\beta}_1^N$

$$\hat{\beta}_1^N \approx \mathbb{E}[YD \mid Z=1] - \mathbb{E}[YD \mid Z=0]$$

To estimate $\textcircled{D} = \Pr(D=1 \mid Z=1) - \Pr(D=1 \mid Z=0)$

$$\mathbb{E}[D \mid Z=1] - \mathbb{E}[D \mid Z=0]$$

Regress D on Z & call it $\hat{\beta}_1^D$

$$\hat{\beta}_1^D \approx \mathbb{E}[D \mid Z=1] - \mathbb{E}[D \mid Z=0]$$

$$= \Pr(D=1 \mid Z=1) - \Pr(D=1 \mid Z=0)$$

$$\frac{\hat{\beta}_1^N}{\hat{\beta}_1^D} \approx \mathbb{E}[Y(1) \mid \text{Compliers}]$$

Imbens & Angrist & Rubin (1996)